Sound transmission through opened windows

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Abstract

Two different models have been derived in order to compute sound transmission through opened windows. The first model is a hybrid model which combines modal, geometrical, wave and integral approaches. The second model is a multi-domain BEM approach which is used as a reference. Comparisons have also been made with published measurements. Numerical aspects are discussed. SEA computations are used to compute structure born transmission in order to verify the predominance of acoustical transfer through the windows.

Keywords: Hybrid model; Opened windows

1. Introduction

In the past decades urban populations have regularly increased. In 2007, the symbolic figure of 50% of humanity leaving in cities has been reached. A large percentage of this population lives in elevated buildings. It is no mystery that sound insulation varies greatly between countries depending on the quality of building constructions and isolation techniques. However, one often forgets that even the best sound insulation will be of little help in summer time or in southern countries where windows are frequently opened. In the case of two apartments with opened windows on a common façade, one expects noise to be mostly transmitted through the windows and along the façade. The need of numeric methods capable of handling such situations has motivated this work.

To the author’s knowledge, little predictive work has been published on this subject. One can mention the study of Dashen and Xijing [1] which presents a simple analytical formula for the computation of noise radiated by an opened window as well as measured sound transmitted between two adjacent rooms. The authors compare structure borne transfer and acoustic transfer when the windows are closed or opened. Their results show that dominant transfer occurs through the opened windows. The authors also propose the use of an external baffle perpendicular to the façade, acting as a noise barrier between both rooms, in order to reduce the external sound transfer.

The work published by Fothergill and Hargreaves [2] is based on measurements of sound transfer through windows which are gradually opened and positioned at variable positions with respect to the common wall between two rooms. The influence of the position is only important when both windows are separated by less than one meter.

The present problem involves only indirect acoustic transmission as the sound waves leave the source room and are diffracted along the edges of the source window. Transmission along the façade will be diffracted again on the receiver window. Therefore the use of a geometrical approach would require modelling multiple reflexions inside rooms and double diffraction effects on the façade in order to be able to reproduce the transmission paths.

On the other hand, precise numerical approaches such as the boundary element method (BEM) can handle such aspects but can hardly deal with whole buildings over large frequency ranges. Nevertheless, BEM has been used as a reference approach; it uses a multi-domain formalism in a simplified situation where the façade is replaced by an infinite baffle thus neglecting diffraction at building edges.
and reflection on the ground. Both source and receiver volumes are modelled. Consequently, the present BEM model considers three domains: external baffled medium and two rooms – unbaflled media.

Kawai et al. [3] also present a BEM approach. Their model is a simplified multi-domain BEM approach for two openings in an infinite baffle. The volumes are not modelled but a set of plane waves excites the source opening so that diffuse field is assumed and modal room behaviour is neglected. Comparisons with measurements and computations from this author are presented.

A faster approach than BEM is here proposed. It is based on a paper by Kropp et al. [4] which presents a hybrid model for balconies. The balcony is a rectangular volume with a rectangular opening placed in an infinite baffle and an external point source is considered. The problem here addressed can be viewed as a succession of two similar problems: (1) an acoustic source inside a rectangular volume radiates noise which is (2) the input acoustic field to Kropp’s model for the second rectangular volume. Thus the problem of sound transmission between two lodgings with opened windows can be modelled by generalizing Kropp’s model for a source within a rectangular volume and by considering two consecutive problems. The model has been even further generalized by considering a source image technique in order to compute the external acoustic field thus allowing the introduction of the ground and of a nearby façade.

The paper is organized as follows: in Section 2, the hybrid model is presented. The BEM model is briefly presented in Section 3. Both models are compared in Section 4. Results from the hybrid model are then compared to Kawai’s results in Section 5. The last part of the paper presents more results with some discussions; in particular a comparison of acoustical and structure borne transfers is showed in order to justify the assumption that the former is dominant.

2. The hybrid model

As mentioned in Section 1, the model here proposed is a generalization and adaptation of an existing model for balconies [4]. Fig. 1 represents the whole situation here considered: two rectangular rooms \( (V_1 \text{ and } V_2) \) with rectangular openings \( (S_{o1} \text{ and } S_{o2}) \) are set in a semi infinite plane façade. An acoustic monopole is placed in volume \( V_1 \). The sound is transmitted to volume \( V_2 \) through the external medium \( \Omega \) which is bounded by a vertical baffle at \( x = 0 \) and eventually by a ground or a ground and a second building.

Fig. 2 shows the two separate problems \( P_1 \) and \( P_2 \) successively considered. First, in problem \( P_1 \), the source volume \( V_1 \) is considered alone. The acoustic pressure is sought either in \( V_1 \) or at any point inside \( \Omega \), in particular on \( S_{o2} \). Problem \( P_2 \) considers volume \( V_2 \) alone; the excitation, which is the output of problem \( P_1 \), comes from \( \Omega \) and enters volume \( V_2 \) through \( S_{o2} \). This second problem is similar to the problem solved by Kropp et al. [4] in the case of a point source outside a balcony (see Fig. 3).

2.1. Kropp’s model for balconies (problem \( P_2 \), external source)

A room (or balcony) of dimensions \( L_x, L_y, L_z \) is considered. A local coordinate system is used in the volume. The
wall at \( x = 0 \) is set in an infinite plane (the façade) – semi infinite when the ground is modelled – and has an opening of dimensions \( D_x, D_z \). The thickness of the wall in the opening is not modelled. Each wall has a constant admittance \( \beta \), named \( \beta_{11}, \beta_{22}, \beta_{21}, \beta_{12} \), respectively for \( x = 0, x = L_x, y = 0, y = L_y, z = 0, z = L_z \). The time convention is \( e^{j\omega t} \). One should also say that Kropp et al. mention the possibility to extend their model to more complex situations by adding the ground plane by means of source image terms.

The sound field inside the volume is described as a combination of a modal approach across the width of the room \((y,z)\) plane and two waves propagating perpendicular to the opening \((+x\text{ and }-x\text{ directions})\) associated to each mode \( mn \):

\[
P_R(x,y,z) = \sum_{mn} A_{mn} c_{mn}(y,z) e^{-ik_{mn}x} + \sum_{mn} B_{mn} c_{mn}(y,z) e^{ik_{mn}(x-L_x)}
\]

(1)

For rigid side walls \((\beta_y = \beta_z = 0)\):

\[
c_{mn}(y,z) = c_m(y) \cdot c_n(z), \quad c_m(y) = \cos\left(\frac{m \cdot \pi \cdot y}{L_y}\right),
\]

\[
c_n(z) = \cos\left(\frac{n \cdot \pi \cdot z}{L_z}\right),
\]

\[
k_{mn}^2 = k^2 - k_y^2 - k_z^2, \quad k_y = \frac{m \pi}{L_y}, \quad k_z = \frac{n \pi}{L_z},
\]

\(k = \omega/c\) is the acoustic wavenumber.

For non rigid walls, a different local coordinate system is employed. The room now extends between \( y = -a \) and \( y = +a, z = -b \) and \( z = +b \) where \( L_y = 2a \) and \( L_z = 2b \). The expressions are only given for the \( y \) component since there are similar in the \( z \) direction.

The following expressions can be found in [5]. When the walls are not rigid and given by a specific admittance \( \beta_0 = \rho c \beta \), the wavenumber in the \( y \) direction is no longer expressed as \( \left(m \pi/L_y\right) \) but becomes complex. We consider a mean admittance for the two parallel \( y \) walls \( \beta = (\beta_{11} + \beta_{12})/2 \), and since the coordinate system is centred one must distinguish odd and even modes.

\[
c_m(k_y \cdot y) = \begin{cases} \cos(k_y \cdot y) & \text{for } m \text{ even} \\ \sin(k_y \cdot y) & \text{for } m \text{ odd} \end{cases}
\]

In order to respect the boundary condition at \( y = -a \) and \( y = +a \) one must solve the following equations:

\[
k_y \cdot \tan(k_y) + B = 0 \quad \text{for } m \text{ even}
\]

\[
k_y \cdot \cot(k_y) - B = 0 \quad \text{for } m \text{ odd}
\]

where \( B = jak_0 \).

A minimization technique – the simplex method- has been employed to obtain the values of \( k_y \). At a given frequency and for a given mode the value of \( k_y \) for rigid conditions is used as a starter ensuring rapid identification.

The acoustic velocity normal to the walls, at \( x = 0 \) and \( x = L_x \) is obtained as \( v_y = \pm v_i = \frac{\partial p}{\rho c} \).

The external acoustic pressure at any point \( M \) in the external domain \( \Omega \) is written by means of the Rayleigh integral carried over the opening

\[
P(M) = P_{\text{incident}} + \int_{S_o} V(\mathbf{Q}) \cdot G(M, Q) \cdot dS(\mathbf{Q})
\]

(2)

where \( G \) is the Green function outside the façade. In the case of an infinite rigid façade and since \( Q \) points are set in the façade, \( G \) writes

\[
G(M,Q) = \frac{e^{-ikR}}{2\pi R} \quad R = ||MQ||
\]

The presence of the ground is introduced by using an image term as

\[
G(M,Q) = \frac{e^{-ikR}}{2\pi R} + \frac{\Gamma e^{-ikR}}{2\pi R}
\]

(3)

where \( R \) is the distance \( ||MQ|| \), \( M' \) being the image of \( M \) with respect to the ground, \( \Gamma \) is the reflexion coefficient which is evaluated by using a plane wave approximation. When a nearby building is added, \( G \) is computed by means of a source image technique which is particularly adapted and easily programmed for this simple situation with only three reflecting planes as illustrated in Fig. 4; this sub-problem being then two dimensional since the \( y \) coordinate does not come into play. In Fig. 4 the façades are drawn as semi infinite but could easily be replaced by finite ones. The finite length would have no influence on the source image solution since all waves are directed upwards and can not be reflected back. However diffraction could be added by means of the GTD technique [6]. Previous computations [7], where a source image technique has been compared to BEM computations, have shown that the diffracted waves have little influence in this case.

The modal coefficients \( A_{mn} \) and \( B_{mn} \) are obtained by writing the boundary conditions at \( x = 0 \) (the front wall containing the opening) and \( x = L_x \) (the rear wall).

For a number \( MN \) of modes \( 2MN \) unknowns \((A_{mn} \text{'s} \text{ and } B_{mn} \text{'s}) \text{ must be determined. Both } x = 0 \text{ and } x = L_x \)
rectangular surfaces are discretized, each into $MN$ rectangular elements and the boundary conditions are written at the centre of each element:

(i) At all nodes on the rear and front wall the condition $V_d/P = \beta_v$ is written. Imposed velocities at $x = L_x$ can also be considered [13].

(ii) At every node in the opening the condition of continuity of acoustic pressure between $\Omega$ and $V_2$ is employed (Eqs. (1) and (2) equated).

2.2. Problem $P_1$ (internal source)

This problem is similar to the previous one. We still have a rectangular volume with an opening $S_o$ set in an infinite baffle. The point source is now placed inside the volume and we mean to compute the acoustic pressure incident on the second opening. Therefore the only differences with the previous problem are:

(i) in Eq. (2) the $P_{\text{inc}}$ term must be suppressed so that now in $\Omega$

$$P(M) = \int_{S_o} V(Q) \cdot G(M, Q) \cdot dS(Q)$$

(ii) in Eq. (1) a source term must be added inside volume $V_1$

$$P_{\text{int}} = P_R + P_S$$

The expression considered for the source term $P_s$ is taken as the pressure of a monopole set in an infinite rectangular duct; an expression can be found in Morse & Ingard [5] as

$$P_s(x, y, z) = \sum_{mn} \Omega_{mn}(y, z; y_S, z_S) e^{-ik_{mn}|x-x_S|}$$

$$x = -i/2S \quad S = L_y \cdot L_z$$

$\Omega_{mn} = c_{mn}(y, z)c_{mn}(y_S, z_S)$, $A_{mn} = 1/A_{mn}A_{mn}$

$A_i = 1$ if $i = 0, 2$ otherwise

In (5), the term $P_R$ is the complementary pressure related to the end conditions ($x = 0$ and $x = L_x$). $P_R$ is expressed in Eq. (1). $P_S$ models waves propagating from the monopole in the room whereas $P_R$ models waves propagating from the end walls at $x = 0$ and $x = L_x$.

2.3. Global problem $P_1 + P_2$

The global problem is simply solved by first solving problem $P_1$ and by computing the incident acoustic pressure at all nodes of opening $S_o$ by means of Eq. (3). Problem $P_2$ is solved in a second stage. This is a decoupled procedure which assumes that the second volume does not influence the solution of problem $P_1$. Comparisons with a fully coupled BEM approach, reported in Section 4, show that this assumption is verified in practical situations.

2.4. Numerical aspects

Special care must be taken when writing the continuity condition on the opening since singular terms arise when $M = Q$ in (2). A semi analytical integration is here employed [8].

The evaluation of the integral term in (2) can be improved by using a sub meshing of $S_o$ so that although the velocity $V$ is still evaluated at the $MN$ nodes, $G$ is evaluated at a multiple number of positions $MN$. Numerical simulations have confirmed this improvement and good results have been obtained with $MN' = 16MN$ (grid spacing divided by four).

The selection of modes employed for the modal calculation is evaluated automatically. At a given frequency all modes corresponding to progressive waves must be considered and a chosen number of non progressive modes is selected with several possible criteria. Calculations have shown that a too stringent modal basis might lead to erroneous results. Thus, in conclusion, both the number of modes and the degree of refinement in evaluating the integral component in Eq. (1) are tuneable.

The computation of the modal expressions in a given volume leads to a matrix system which is first diagonalized. The determination of the modal coefficients for a given excitation (back-substitution of the diagonalized matrix) is much faster than the previous step so that storing the diagonalized matrices offers a means to study new situations much faster such as modifying the external acoustical field (presence of a ground, of a second building) or the spacing between the two volumes. Therefore, for a given volume, defined by its geometry and wall properties and opening size, a condensed matrix has to be stored for all computed frequencies. One will also note that when both volumes are identical the condensed matrix is only computed for the first volume.

Last, it must be mentioned that in the present derivations the problem of partially opened windows is a priori excluded. However, one might argue that by using reduced openings the case of partially opened windows might be approached. The case of openings of more complex shapes could also easily be considered since the integration surface $S_o$ in (2) and (4) is not restricted to rectangular openings.

3. The BEM model

An existing 3D BEM model has been extended to the case of two opened volumes set in an infinite baffle. This model is here employed as a validation tool. The cases used for validation will therefore include neither the ground nor any nearby building. This BEM model is based on a variational approach originally written for single domain acoustics [8] in 3D. In [9], a 2D version for outdoor sound propagation problems has been derived. This model has
been adapted to 3D [10] and 2D [11] vibro-acoustics problems. In [12], multi-domains have been introduced in the 2D BEM model developed at CSTB for outdoor sound propagation problems in order to study roads below grade and porous asphalt layers under tyres. The underlying equation is the full integral representation in terms of both pressure $P$ and displacement $W$:

$$
c(P)P(M) = \int_{S_0} \left[ \rho \omega^2 W(Q) G(M, Q) - P(Q) \frac{\partial G(M, Q)}{\partial n_Q} \right] dS(Q)
+ \int_{S_o} P(Q) \left[ \rho \omega^2 Y(Q) G(M, Q) - \frac{\partial G(M, Q)}{\partial n_Q} \right] dS(Q)
$$

$$\gamma = 1 \text{ if } M \notin S, \quad \gamma = 1/2 \text{ if } M \in S \text{ at smooth points}$$

(7)

Where $S_0$ denotes acoustic boundaries (the walls inside the volumes) defined by a mobility $Y$, $S_o$ stands for the openings which act as interfaces between outside and inside domains. On $S_o$ both $P$ and $W$ are unknown.

As for the previous hybrid model, the acoustic domain is separated into three domains: two volumes and one baffled infinite external domain. In the three domains the generalized variational expression [10,11] in terms of $P$ and $W$ derived from (7) must be employed. In the external domain, the baffled Green function (3) must be used[12]. Continuity of $P$ and $W$ is written at the interfaces ($S_{o1}$ and $S_{o2}$) when building the global matrix system. This matrix, contrary to single domain BEM, is not fully populated since finite elements of volumes $V_1$ and $V_2$ not included in the openings are not linked.

4. Comparison between models

The program for the hybrid model has been named BANCAL, which stands for Balcony Numeric CALculation, and the BEM code is called MICADO. The purpose of this section is to validate both programs by comparing their results in a well defined situation. Comparisons with scale measurements and computations found in the literature are presented in the next section.

The case of two volumes of identical dimensions is considered. Each volume is 4 m deep ($L_x$), 3 m wide ($L_y$) and 2.7 m high ($L_z$). The openings have dimensions 2 m ($D_y$) × 1.7 m ($D_z$). They are horizontally centred and set 1 m above the floors of each volume. The walls are assumed to be rigid. The floors of both volumes are 4 m apart ($H = 4$ m in Fig. 1). The ground is omitted.

A monopole unit source is placed in the higher volume at $(3.9, 1.5, 0.1)$ near the floor and rear wall. The source is at a central $y$ position so that the problem is symmetric with respect to $y$ in order to reduce the computation time for the BEM calculations. The acoustic pressure is computed at several frequencies on a trajectory centred with respect to the width ($y = 1.5$ m); it is made of three portions defined by

1. In and out of the source volume at $z = 1.5$ m: $x = 4$ m to $x = -0.1$ m (local coordinate system of $V_1$),
2. along the façade: $x = -0.1$ m (downwards)
3. Out and inside the second volume at $z = 1.5$ m, $x = -0.1$ m to $x = 4$ m (local coordinate system of $V_2$).

the trajectory leaves or enters the volumes at $x = 0$ m. This contour is illustrated by a dashed line in the caption of Fig. 5.

Fig. 5 compares the above-defined evolution of the acoustic pressure at 100, 400 and 800 Hz. The comparison is very satisfactory since differences are mostly inferior to 1 dB.
Fig. 6 shows a different comparison by means of the difference of the average sound pressure levels in the source and the receiver volumes, in third octave bands. Again a good comparison can be observed.

Fig. 7 shows pressure maps in the vertical \((x, z)\) plane, again in the central \(y = 1.5\) position. Both models give very similar pictures.

5. Comparison with Kawai’s results

Kawai [3] has proposed a simpler BEM model where only the openings are discretized. The rooms’ walls are not modelled but replaced by a diffuse excitation in the source volume by means of 825 incident uncorrelated plane waves. Measurements have been made on a 1/10 scale model made of two rectangular volumes; diffusers have been added in order to have a better agreement with the model’s assumptions. Openings of different dimensions have been analyzed. The case here considered is that of openings of dimensions \(20 \times 20 \text{ cm}^2\) (real openings \(2 \times 2 \text{ m}^2\)). The two openings are set in a rigid baffle with a variable spacing \(H\). Fig. 8 shows a schematic representation of the configuration studied by Kawai here considered. The two rooms have dimensions \(7 \times 4 \times 6 \text{ m}^3\), the openings are \(2 \times 2 \text{ m}^2\) and \(z\) centred. The lower opening is 0.3 m away from the ceiling of the lower volume; the upper opening is vertically symmetric to the lower one. There is no ground. In his experiments Kawai has placed absorbent layers on the walls thus probably creating low reverberation conditions. However no data are provided for absorption values so that it is difficult to know how to correlate these measurements with actual lodgings conditions.

We define

\[
TR = 10 \cdot \log(W_1/W_2) \\
DL = L_1 - L_2 \approx TR + 10 \cdot \log(A/S)
\]

where \(W_1\) and \(W_2\) are, respectively the powers leaving the source volume and entering the second volume; \(S\) is the area of the openings and \(A\) is the absorption area of the receiving volume. \(L_1\) and \(L_2\) are the mean sound pressure levels in both volumes. Kawai has plotted charts of \(TR(H)\) both measured and computed for several octave bands with good agreement. One must however notice that the actual rooms have large dimensions \((7 \times 4 \times 6 \text{ m}^3)\) more representative of offices than actual lodgings. Large dimensions were appropriate because of the assumption of a diffuse field made by Kawai’s approach.
Relation (8) between $TR$ and $DL$ is based on a diffuse field assumption. Since both W’s and L’s can be computed by the present hybrid approach a comparison of $DL$ and $TR + 10 \cdot \log(A/S)$ is presented in Fig. 9 for three cases of wall absorption defined in terms of normal incidence absorption coefficients $\alpha_n$, respectively of 0.01, 0.10 and 0.20 (corresponding approximately to diffuse absorption coefficients twice as large). Equality (8) appears rather well verified for $\alpha_n = 0.10$ –within 2 dB- but less so for $\alpha_n = 0.01$ or 0.20 with differences which exceed 3 dB. Fig. 9 also shows the increase of sound insulation due to an increase of room absorption.

Plots of $TR(H)$ are given in Fig. 10 at 200 Hz, 500 Hz and 1000 Hz, as measured and computed by Kawai and computed by the present hybrid approach. Results obtained with the hybrid approach are given for two values of room absorption. Results with $\alpha_n = 0.10$ are in better agreement with Kawai’s results as one would expect considering the presence of absorbent material on the walls of the experimental set-up. The object of this comparison was mainly to check that orders of magnitude obtained by the present model agreed at least qualitatively with published measurements.

The effect of room size on the charts of $TR(H)$ is plotted in Fig. 11. In addition to the room dimensions considered by Kawai ($7 \times 4 \times 6$ m$^3$) a second smaller geometry is considered where rooms have dimensions $3 \times 4 \times 2.7$ m$^3$ and openings $2 \times 1.4$ m$^2$, $y$ centred and 1 m above the rooms’ floors. The influence of the room geometry can mostly be seen for the shorter spacing $H$, with differences up to 10 dB; the smaller geometry leads to smaller values of $TR$.

Finally, the effect of a rigid ground is considered by assuming the source room to be at ground level. The effect of a rigid nearby building is also considered. As for the main façade this second building, placed at a distance $D$ from the first façade, is only introduced by means of a semi infinite baffle without diffraction. Fig. 12 compares, for two values of the room separation $H$, values of the sound pressure level difference $DL$ for the reference situation with no ground then with a rigid ground and for two values of $D$ (30, 15 m). The geometry is the same as the $3 \times 4 \times 2.7$ m$^3$ volume of Fig. 11. It clearly appears that $DL$ is reduced when a second façade is added, the shorter $D$ or the larger $H$, the larger the decrease. One also notices that for adjacent volumes ($H = 3$ m) all acoustical results are grouped meaning that, in this case, neither the presence of ground nor of a nearby building has any influence on sound transmitted from one lodging to the next.

Fig. 12 also includes results for structure borne noise obtained by SEA (Statistical Analysis Approach). SEA computations have been made in order to compare sound propagated through the opened windows and sound transmitted by the vibrating walls. The SEA code employed is an SEA code developed at CSTB [14]. Fig. 13 shows the building considered. It has horizontal dimensions of $20 \times 15$ m. The lower level is an underground basement with external walls and floor for which loss factors obtained from measurements are considered in order to account for ground loading [15]. The rooms in the central column on
one of the shorter sides have been considered. A noise source is placed at level 1 (ground floor) and noise levels at levels 2, 3 and 5 are computed relative to the excited volume so that values of DL due to structure borne transfer can be compared to airborne values. SEA results are meaningless below 100 Hz, due to an insufficient number of modes, and are not reported. We notice in Fig. 12, that structure borne attenuation is higher than acoustical values, meaning that we can neglect structural contributions above 200 Hz; this legitimates the present hybrid model based solely on acoustical transfers. Considering the crossing of acoustical and structural plots around 100 Hz one might however suspect that the computed low frequency sound attenuation might require the consideration of structural borne aspects even when windows are opened.

6. Conclusions

The problem of noise transmission between dwellings with opened windows has been studied. A dedicated hybrid model has been developed and validated against a modified BEM model. By mixing several approaches, computation times in the hybrid approach can be greatly reduced. Since a sequential and decoupled methodology has been chosen, computation of rooms’ responses can be made independently of external sound propagation. The model can consider the presence of a nearby building and more complex situations can be analyzed provided that one has the use of a fast computer code such as a ray tracing approach [6,12] to compute the Green function in the external acoustic field. Results have been found to be in good agreement.
with published charts of sound attenuation. Acoustical transmission has also been compared to structure borne sound transmission. It has been shown that above 200 Hz acoustical transfer is sufficient to assess sound isolation through open windows.

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